## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

## U.G. DEGREE EXAMINATION - ALLIED <br> FIRST SEMESTER - NOVEMBER 2023 <br> UMT 1303 - MATHEMATICS FOR STATISTICS - I

Date: 09-11-2023
Time: 09:00 AM - 12:00 NOON
Max. : 100 Marks

| SECTION A - K1 (CO1) |  |
| :---: | :---: |
|  | Answer ALL the Questions $\quad(10 \times 1=10)$ |
| 1. | Answer the following |
| a) | What is the product rule in differentiation? |
| b) | Define a critical point of a function. |
| c) | State Euler's theorem. |
| d) | Describe integration by parts. |
| e) | List any two applications of integral calculus. |
| 2. | Fill in the blanks |
| a) | In implicit differentiation, differentiating both sides of an implicit equation with respect to the ................. variable is the initial step. |
| b) | If the second derivative of a function is positive at a specific point, the function has a local .............. at that point. |
| c) | The partial derivative of a function with respect to a variable means that the $\qquad$ of the function concerning that variable. |
| d) | The integral of $\frac{1}{\sqrt{x}}$ with respect to $x$ is |
| e) | The definite integral of $\frac{1}{x}$ from 1 to e is |
|  | SECTION A - K2 (CO1) |
|  | Answer ALL the Questions $\quad$ (10 x 1 = 10) |
| 3. | Choose the correct answer |
| a) | If $f(x)=\tan x$, what is $f^{\prime}(x)$ ? <br> (i) $\cos ^{2} x$ <br> (ii) $\sec ^{2} x$ <br> (iii) $\cot x$ <br> (iv) $\sin ^{2} x$ |
| b) | Leibnitz theorem provides a formula to find <br> (i) The $n^{\text {th }}$ derivative of a sum of two functions <br> (ii) The $n^{\text {th }}$ derivative of a product of two functions <br> (iii) The integral of a product of two functions <br> (iv) The integral of a sum of two functions |
| c) | What is a partial differential equation? <br> (i) An equation involving only partial derivatives of a function. <br> (ii) An equation involving only ordinary derivatives of a function. <br> (iii) An equation involving both partial and ordinary derivatives of a function. <br> (iv) An equation involving only algebraic expressions. |

d) Which of the following integrals represent integration of a rational algebraic function?
(i) $\int e^{x} d x$
(ii) $\int \frac{1}{x^{2}+1} d x$
(iii) $\int \frac{x^{2}+3 x-2}{x+1} d x$
(iv) $\int \sin x d x$
e) The value of the definite integral $\int_{0}^{\frac{\pi}{2}} \sin x d x$ is
(i) 0
(ii) 1
(iii) $\pi$
(iv) 2
4. State true or false
a) The derivative of a constant multiplied by a function $k . f(x)$ is $k . f^{\prime}(x)$.
b) If a function is concave downwards on an interval, then the second derivative is positive.
c) If a function $f(x, y)$ is independent of the variable $y$, then $\frac{\partial f}{\partial y}=y$
d) The integral of a function over an interval cannot be negative.
e) If $F(X)$ is an antiderivative of $f(x)$, then $\int_{a}^{b} f(x) d x=F(b)-F(a)$.

## SECTION B - K3 (CO2)

## Answer any TWO of the following

5. Apply Leibnitz theorem to compute $n^{\text {th }}$ derivative of $x^{2} e^{5 x}$.
6. Produce $y_{n}$ when $y=\frac{3}{(x+1)(2 x-1)}$ by resolving into partial fractions.
7. $\quad$ Determine $\int \frac{x^{24}}{x^{10}+1} \mathrm{dx}$.
8. Using partial fraction method, compute $\int \frac{2 d x}{(1-x)\left(1+x^{2}\right)}$

## SECTION C - K4 (CO3)

## Answer any TWO of the following

$(2 \times 10=20)$
9. Examine Euler's theorem when $u=x^{3}+y^{3}+z^{3}+3 x y z$.
10. Illustrate the theorem that $\frac{\partial^{2} u}{\partial x \partial y}=\frac{\partial^{2} u}{\partial y \partial x}$ when u is equal to $\log \left(\frac{x^{2}+y^{2}}{x y}\right)$.
11. Test any three properties of definite integral.
12. Determine $\int \frac{d x}{(3+x) \sqrt{x}}$

## SECTION D - K5 (CO4)

## Answer any ONE of the following

$(1 \times 20=20)$
13.
(i)If $y=\sin \left(m \sin ^{-1} x\right)$, Defend that $\left(1-x^{2}\right) y_{2}-x y_{1}+m^{2} y=0$.
(ii) Predict $n^{\text {th }}$ differential coefficient of $x^{2} \log x$.
14. (i) Evaluate $\int(3 x-2) \sqrt{x^{2}+x+1} d x$.
(ii) Compute $\int \frac{x+4}{6 x-7-x^{2}} d x$

## SECTION E - K6 (CO5)

## Answer any ONE of the following

$(1 \times 20=20)$
15. $\quad$ Assess the maximum and minimum values of the function $2 x^{3}-3 x^{2}-36 x+10$.
16. (i) Compile reduction formula for $\int \cos ^{n} x d x$ and hence evaluate $\int_{0}^{\frac{\pi}{2}} \cos ^{8} x d x$.
(ii) Test that $\int_{0}^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}}+(\cos x)^{\frac{3}{2}}} d x=\frac{\pi}{4}$ (10 marks)

